S.No. 8046

24DPMA01

(For the candidates admitted from 2024-2025 onwards)

M.Sc. DEGREE EXAMINATION, AUGUST 2025

First Semester

Maths

ALGEBRAIC STRUCTURE

Time : Three hours — Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions

1. Define normalizer.

2. Define P-Sylow subgroup of G.

3. Define the internal direct product of G.

4. Define the R-Module.

5. Define similar transformation.

6. Determine the index of nil—potent of T.

7. Write down the Jordan Canonical form.

8. Describe the companion matrix of $f(x)$.

9. Define trace of 'A'.

10. Define unitary transformation.

PART B — ($3 \times 5 = 15$ marks)

Answer any THREE questions out of Five questions

11. Prove that $N(a)$ is a subgroup of G.

12. Suppose that G is the internal direct product of $N_l,\ldots,N_k$ For $i \neq j$, $N_i \cap N_j = (e)$, and if $a \in N_i, b \in N_j$, then prove that $ab = ba$.

13. Show that if two nilpotent linear transformations are similar if and only if they have the same invariants.

14. If two matrices A, B in F_n are similar in K_n where K is an extension of F. Then prove that A and B are already similar in F_n.

15. 1f F is of characteristic 0 and if S and T, in $A_F(v)$, are such that $ST - TS$ commutes with S, then prove that $ST - TS$ is nilpolent.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions.

16. (a) If $0(G) = p^n$ where p is a prime number, then prove that $Z(G) \neq (e)$.

Or

(b) State and prove first part of Sylow's theorem.

17. (a) Let R be a Euclidean ring, then prove that any finitely generated R–module M, is the direct sum of a finite number of cyclic sub modules.

Or

(b) Let $G = S_n$, where $n \geq 5$; then $G^{(K)}$ for $K = 1,2,....$ contains every 3 - cycle of S_n.

18. (a) If $T \in A(V)$ has all its characteristics roots in F, then there is a basis of V in which the matrix of T is triangular.

Or

(b) Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$, where V_1 is a subspace of V.

19. (a) For each $i = 1,2,....k$, $V_i \neq (0)$ and $V = V_1 \oplus V_2 \oplus ... \oplus V_K$. Prove that the minimal polynomial of T_i, is $q_i(x)^{li}$.

Or

(b) Let V and W be two vector space over F and suppose that i is a vector space isomorphism of V onto W. Suppose that $S \in A_F(V)$ and $T \in A_F(W)$ are such that for any $v \in V$, $(vS)\ \psi = (v\ \psi)T$. Then prove that S and T have the same elementary divisors.

20. (a) Prove that for all $A, B \in F_n$

(i) $(A')' = A$

(ii) $(A + B)' = A' + B'$

(iii) $(AB)' - B'A'$

Or

(b) If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, then prove that $T = 0$.